MATHEMATICAL ANALYSIS OF A LAYER EXTRATION METHOD FOR SEPARATING CLAY-SIZE MATERIAL FROM SOILS

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ABSTRACT

This paper presents a layer extraction method for obtaining a representative sample of clay-size material from a soil suspension. This method is especially advantageous in preparing a sample of relatively large size, and the procedure is simpler than that of the decantation method, a comparable method of extraction.

In the layer method, the position and the thickness of the layer to be extracted from a soil suspension can be varied and one or more extractions can be made. The effect of these variables on the size composition and the quantity of the extracted material is analyzed mathematically. Based upon such an analysis, the proper position and thickness of the layer and the desirable number of extractions can be determined to suit the requirements of a particular test.

INTRODUCTION

Properties of common soils depend to a large extent upon the amount and characteristics of the clay-size material contained in the soils. For this reason, it is often desirable to separate the clay-size material from a natural soil and to study the characteristics of the material separated.

Whereas different types of elutriators (Bayer, 1948, p. 41; Handy, 1953, p. 136) have been used for making the required separation, sedimentation methods are in common use, especially for separating materials finer than 0.002 mm. The procedure for making the separation by sedimentation methods includes dispersion of a soil sample (Wintemyer, 1948; Chu, 1953; Bouyoucos, 1951), sedimentation of the suspended soil particles by gravity or by centrifugation, extraction of the required clay-size material from the soil suspension (Seay, 1948; Havens, 1948; Havens, 1950; Hauth, 1951), and drying, if necessary, of the extracted soil suspension. This paper presents the theory of a layer method of extraction.

While this method was developed primarily for separating the clay-size material from soils, it can be used for separating any size fraction from any material whose component parts can be differentiated by sedimentation methods. Various types of apparatus may be used in the layer extraction method. Experiments with extraction apparatus are expected to be made in the future.
REVIEW OF EXTRACTION METHODS

The method of extraction used has an important bearing on the size composition of the clay-size material separated from a soil sample. Usually it is desirable that the gradation of the material collected be the same as the gradation of the clay-size material in the dispersed sample. The extraction procedure should also be relatively simple. These two requirements can be used as criteria in evaluating different extraction methods.

**Decantation Method**

The method herein referred to as the *decantation method* is commonly used when a comparatively large quantity of clay-size material is needed. The procedure is as follows:

A soil sample is dispersed in distilled water and the resulting suspension allowed to settle in a container. After a given period of sedimentation, the portion of soil suspension containing soil particles smaller than a specified size is removed from the container by decantation or siphoning. The container is then refilled with distilled water to the original level, the diluted soil suspension allowed to settle after shaking, and the extraction process repeated. This procedure is continued until the portion to be removed contains few or no suspended soil particles. The several withdrawals are combined to obtain a representative sample of the desired clay fraction.

The decantation method is satisfactory when the proper number of decantations are made; if not, the gradation of the material collected will not be representative. Perhaps the main disadvantage of this method is that it is exceedingly time-consuming, both as to the large number of repeated decantations required and to the large quantity of water that must be separated from the clay material by drying or centrifuging. Another disadvantage is that the repeated decantations tend to magnify experimental errors.

**Other Methods**

A pipette may be used for extraction purposes (Collini, 1943). After a given sedimentation period, the pipette is lowered into the soil suspension to a predetermined depth and a small quantity, usually 5 to 25 ml, is removed with the pipette. Although a representative sample can be obtained in this manner, the use of the pipette is limited to experiments requiring only a small amount of clay-size material.

The extraction of clay-size material from a soil suspension can be accomplished with a continuous-type supercentrifuge. The supercentrifuge serves to accelerate sedimentation as well as to separate the desired clay fraction. This method is especially useful for separating clay material into different particle-size range fractions. The use of the supercentrifuge has been discussed by Baver (1948, p. 39).

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1 Possibly 20 to 30 with fine-grained soils.
The principle of the supercentrifuge method is similar in many respects to that of the decantation method. While long sedimentation periods can be eliminated by use of the supercentrifuge, the repeated treatments necessary for complete separation and the necessity for collecting the clay-size material from a large amount of suspension make the method time-consuming. Except for those laboratories which do frequent testing, the expensive and comparatively complex equipment requirements are a decided disadvantage in using the supercentrifuge method.

**LAYER EXTRACTION METHOD**

The layer extraction method was developed for the purpose of obtaining representative samples of the material finer than a given size by a simple and comparatively short procedure. This method is suitable for separating relatively large quantities of clay-size material from soils. Although the layer method can be used to fractionate further the clay-size material separated, it probably is advantageous to use a supercentrifuge.

![Figure 1. Soil suspension removed by layer method and decantation method of extraction.](image)

Among the different extraction methods, the decantation method is perhaps most comparable with the layer method. Figure 1 illustrates the difference in the position of suspension removed in the decantation and layer methods. Level A-A in the figure represents the level above which no particles coarser than the maximum particle size of the desired clay fraction will remain in suspension after a given sedimentation period. In the decantation method of extraction, the soil suspension above level A-A is removed. The portion of soil suspension removed in the layer method is a layer bordering or embracing level A-A. When the layer method is used, repeated removals are not required to obtain a representative sample; but if the amount of material collected from the first extraction is insufficient, additional extractions from the remaining soil suspension can be made. After the first extraction, the soil suspension remaining in the container is composed of the portion below level A-A and the portion above level L-L. This remaining suspension can be reshaken and another layer removed after sedimentation.
The diagrams in Figure 2 illustrate the basic theory of the layer method. Only five particle sizes are contained in the dispersed hypothetical sample. The distribution of the five particle sizes in the suspension is shown schematically in the diagrams, which show that all particles are uniformly dispersed throughout the entire depth of the suspension before settling begins.

If size 3 is the maximum size in the desired fraction, the ratio of the three sizes contained in the fraction is 1:1:1. The distribution of the suspended particles after a given period of sedimentation and the position of the layer to be extracted by the layer method are shown in Figure 2(b). Note that the ratio of particle sizes 1, 2, and 3 in the layer is also 1:1:1. A representative sample can thus be obtained from a single extraction.

MATHEMATICAL ANALYSIS

The theory of the layer method of extraction can be verified either by laboratory experiments or by mathematical analysis. The latter method is used in this presentation. To avoid complex mathematical expressions, the analysis is presented by numerical examples. All equations used in the numerical examples are derived in Appendix I, and their application in computing the theoretical gradation of materials obtained is demonstrated in Appendix II. In these computations the material collected from an extracted layer is assumed to be the same as the material contained in the layer prior to extraction.

A hypothetical soil with gradation as shown in Table 1 and Figure 3 was used in the numerical examples. In separating the clay-size material from this soil, it is assumed that a 150 g sample is dispersed in distilled water and the resulting suspension put in a container, shaken, and allowed to settle by gravity. It is further assumed that the depth of the suspension in the container is 21.00 cm.

If the material finer than 0.00200 mm is the clay fraction desired, computations can be made to determine level A-A (see Figure 1) above
which no particles coarser than 0.00200 mm will remain in suspension after a given period of sedimentation. According to Stokes’ law,\(^1\)

\[
h = \frac{980 (G-G_i) TD^2}{30 n}
\]

where 
- \(h\) = distance in centimeters through which soil particles settle 
- \(G\) = specific gravity of soil particles 
- \(G_i\) = specific gravity of the suspending medium, in this case water 
- \(T\) = time in minutes, period of sedimentation 
- \(D\) = particle diameter in millimeters 
- \(n\) = absolute viscosity of the suspending medium in poises.

If \(G = 2.730\) 
\(G_i = 0.998\) (water at 67°F) 
\(T = 360\) min.

\(^1\)Assumptions and limitations of Stokes’ law are discussed in detail by Baver (1948).
Then \( h = 2.00 \times 10^6 D^2 \) (2)  
In this case \( D = 0.00200 \) mm  
and from equation (2) \( h = 8.00 \) cm  
Level A-A is therefore 8.00 cm below the surface of the soil suspension.

Among the assumptions above, those relating to the method of sedimentation and to the gradation of the hypothetical soil should be further clarified. Although gravity sedimentation is assumed in the numerical examples, the layer method can be used as well for extracting clay-size material from a soil suspension in which the sedimentation is accelerated by centrifugation. Figure 3 shows that the gradation of the hypothetical soil contains no material finer than 0.00010 mm and that the segment of the particle-size accumulation curve representing the fraction finer than about 0.0025 mm is nearly a straight line. Actually a soil having almost any gradation could have been selected as the hypothetical soil for the mathematical analysis, but the analysis is greatly simplified by using the hypothetical soil. In general, conclusions drawn from the analysis will hold true for all soils.

In the layer method of extraction, the position and the thickness of the layer to be extracted from a soil suspension can be varied. In addition, one or more extractions can be made to obtain a sample of the desired clay fraction. These variables will be considered in the verification of the layer method theory.

Four typical cases representing possible variations in the position and thickness of the layer are given in Table 2. The term **limiting particle size** used in the table refers to the maximum particle size occurring at a specific level in the soil suspension after a given period of sedimentation.

**Position of Layer**

The gradation of the clay-size materials obtained from layers positioned as in Cases I, II, and III can be determined from equations presented in Appendix I. A computation which illustrates this is given in Appendix II. Computed gradations for the three cases are shown as accumulation curves in Figure 4. The gradation in Case III is closest to the desired gradation, but the material obtained from this layer contains a small amount of particles coarser than 0.00200 mm, the maximum particle size of the desired clay fraction. Case I material, on the other hand, meets the maximum particle-size requirement but is not as satisfactory with respect to overall gradation. It is impossible to select a layer position which will give the best results without consideration of both the thickness of the extracted layer and the method of extraction because they affect the gradation.

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1 The term **desired gradation** refers to the gradation of the desired clay fraction, in this case, the minus 0.00200 mm material.
### Table 2. — Special Cases of the Layer Method

<table>
<thead>
<tr>
<th>Position of the layer with respect to Level A-A&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Top of the Layer</th>
<th>Bottom of the Layer</th>
<th>Thickness of the layer, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h_g', \text{ cm.} )</td>
<td>Limiting Particle Size, mm</td>
<td>( h_g', \text{ cm.} )</td>
</tr>
<tr>
<td>Case I Above level A-A</td>
<td>4.50</td>
<td>0.00150</td>
<td>8.00</td>
</tr>
<tr>
<td>Case II Below level A-A</td>
<td>8.00</td>
<td>0.00200</td>
<td>12.50</td>
</tr>
<tr>
<td>Case III Partly above and partly below level A-A</td>
<td>6.12</td>
<td>0.00175</td>
<td>10.12</td>
</tr>
<tr>
<td>Case IV Same position as Case III</td>
<td>4.50</td>
<td>0.00150</td>
<td>12.50</td>
</tr>
</tbody>
</table>

<sup>1</sup> See Figure 1.
Table 3.—Comparison of Clay-Size Material Extracted by Layer and Decantation Methods

<table>
<thead>
<tr>
<th>Particle Size, mm</th>
<th>Percent by Weight in the Desired Clay Fraction</th>
<th>Material Obtained by Layer Method1</th>
<th>Material Obtained by Decantation Method1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finer Than</td>
<td>Coarser Than</td>
<td>Weight, g</td>
<td>Percent</td>
</tr>
<tr>
<td>0.00225</td>
<td>0.00200</td>
<td>0.12</td>
<td>1.0</td>
</tr>
<tr>
<td>0.00200</td>
<td>0.00175</td>
<td>0.39</td>
<td>3.4</td>
</tr>
<tr>
<td>0.00175</td>
<td>0.00150</td>
<td>0.58</td>
<td>5.1</td>
</tr>
<tr>
<td>0.00150</td>
<td>0.00100</td>
<td>1.57</td>
<td>13.8</td>
</tr>
<tr>
<td>0.00100</td>
<td>0.00060</td>
<td>1.93</td>
<td>16.9</td>
</tr>
<tr>
<td>0.00060</td>
<td>0.00030</td>
<td>2.63</td>
<td>23.0</td>
</tr>
<tr>
<td>0.00030</td>
<td>0.00010</td>
<td>4.20</td>
<td>36.8</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td>100.0</td>
<td>11.42</td>
</tr>
</tbody>
</table>

1 In both the decantation method and the layer method, the data refer to the material obtained from a single extraction.

Figure 4.—Particle-size accumulation curves showing differences in gradation of clay-size material obtained from layers positioned as in Case I, II, and III (see Table 2).

Thickness of Layer

If the position of the layer is properly chosen, the thinner the layer the more closely will the gradation of the material obtained approach the desired gradation. This is illustrated by comparing Cases III and IV (Table 2). Figure 5 shows the desired gradation as well as the computed gradations of materials obtained from layers positioned as in Cases III and IV. Note that the material from the thinner layer, Case III, has the more desirable gradation.

In practical application of the layer method, the choice of layer thickness would also be influenced by the quantity of clay-size material desired. If thinner layers are chosen, more time and effort will be necessary to obtain a sample of the desired quantity.
Although repeated extractions are not required in using the layer method, three or four extractions from a soil suspension may be necessary to obtain a sample of larger size. The difference in gradation of the materials obtained from repeated extractions will be very small and perhaps can be considered within the range of experimental error. This is illustrated by the following comparison of the gradations of the clay-size material obtained from the first and second extractions of a layer positioned as in Case I.

Since the original depth of the hypothetical soil suspension was assumed to be 21.00 cm, the removal of the first 3.50 cm layer (see Table
2) will leave 17.50 cm of suspension in the container. This remaining suspension is re-shaken and after a given sedimentation period another 3.50 cm layer is withdrawn. The theoretical particle-size accumulation curve of the soil in the suspension after the first extraction is shown in Figure 6. By using this particle-size accumulation curve and the equations in Appendix I, the gradation of the material from the second extraction can be determined.

Portions of the computed particle-size accumulation curves representing the clay-size material obtained from the two extractions are plotted in Figure 7. It is apparent that the gradation of the material collected from the second extraction more nearly approaches the desired gradation, but the difference between the two curves is very small.

![Graph showing particle-size accumulation curves](image)

**Figure 7.**—Upper portion of particle size accumulation curves showing differences in gradation of clay-size material in first and second extractions from Case I layer (see Table 2).

**COMPARISON OF LAYER METHOD AND DECANTATION METHOD**

Inasmuch as the layer method is considered more similar to the decantation method than to any other, a comparison of these two methods is made for further evaluation of the former method. As have been discussed, the criteria for evaluation are whether or not the extracted sample is typical, and the amount of time and effort needed to obtain a desired quantity of the sample.

The representativeness of the material obtained from a single extraction by each method is illustrated by the schematic diagrams in Figure 2. In discussing the layer method of extraction, it was pointed out that the ratio of particle sizes 1, 2, and 3 is 1:1:1 both in the desired fraction and in the material obtained by the layer method. Figure 2(b) shows
that in the decantation method, the size ratio is 3:2:1 in the material obtained by a single extraction. The material thus obtained is, therefore, not representative of the fraction desired.

A numerical example may be used to compare further the samples obtained from a single extraction by the two methods. In this example assumptions such as the gradation of the hypothetical soil, the height of the soil suspension, and the size range of the clay material to be separated are the same as in the previous examples.

Both the position and thickness of the layer of soil suspension extracted by the layer method can be varied. For comparison, the layer of Case III (Table 2) will be assumed. The use of the equations in Appendix I for determining the size composition of the material in a layer is shown in Appendix II. In the decantation method of extraction, the soil suspension above level A-A (Fig. 1) is removed. For the assumed conditions, level A-A is 8.00 cm below the surface of the suspension (see section on mathematical analysis). The size composition of the extracted material can also be determined by use of the equations in Appendix I.

The size composition of the materials obtained by the two methods of extraction shown in Table 3 and in Figure 8 indicate that a nearly representative sample can be obtained from a single extraction by the layer method. But, since the gradation of the material obtained from a single extraction by the decantation method differs considerably from the desired gradation, repeated withdrawals are necessary to obtain a representative sample by the decantation method.

**Figure 8.**—Particle-size accumulation curves showing differences in gradation of clay-size material obtained from single extractions by the layer method and by the decantation method.
The large number of repeated withdrawals and the amount of water mixed with the clay-size material owing to repeated dilutions make extraction by the decantation method a time-consuming process. The layer method simplifies the process by eliminating the necessity for repeated withdrawals and continuous dilutions without sacrificing the representativeness of the sample obtained. The possible magnification of experimental errors due to the large number of repeated withdrawals may also be avoided in the layer method of extraction.

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REFERENCES CITED


APPENDIX I

Derivation of Equations

The equations presented in this appendix may be used to determine the size composition of materials obtained by the layer and the decantation methods of extraction. The equations are derived on the assumption of gravity sedimentation, although the sedimentation of suspended soil particles can be either by gravity or by centrifugation.
In the layer and the decantation methods of extraction, the procedure involves the removal of a portion of soil suspension from a container. Different kinds of apparatus may be used for this purpose, but any method used causes some disturbance in the desired portion and the suspension adjacent to it. The material obtained by extraction therefore may differ from that actually contained in the desired portion prior to extraction. In the mathematical analysis presented herein, such differences are assumed to be negligible.

Assume that a soil sample is properly dispersed and allowed to settle by gravity in a container as in Figure 9(a). Before sedimentation, particles of all sizes are uniformly distributed throughout the suspending medium. After a given sedimentation period, particles of different sizes will have settled through different distances which can be determined by Stokes' law. For a given sample, a given sedimentation period, and a given temperature during sedimentation, Stokes' law as expressed by equation (1) can be written as

$$h = KD^2$$

where $h =$ distance through which soil particles settle,

$D =$ particle diameter,

$K$ is a constant.

Assume that particles of different sizes are at the surface (level T-T) of the soil suspension before sedimentation. Then the position of each of these particles after a given sedimentation period can be determined by computing its settling distance according to equation (3). The relative positions of particles of various diameters after a given sedimentation period are shown in Figure 9(b).

Figure 9(a) shows a layer of soil suspension (between levels B-B and C-C) which will be extracted from the container after a given sedimentation period. Note that no particles with diameter greater than $D_b$ remain in suspension above level B-B and none with diameter greater than $D_c$ remain in suspension above level C-C at the instant of extraction. It has been mentioned that before sedimentation all particles are uniformly distributed throughout the suspending medium. The amount of soil obtained from the layer at the end of a given sedimentation period therefore will be smaller than the amount of soil suspended in the same layer before sedimentation.

The particle-size accumulation curve is conventionally plotted on semilogarithmic paper with the particle diameter on the logarithmic scale. This accumulation curve of any soil can be approximated by several straight line segments. The general equation of

$$P = m \log D + K_1$$
where \( P \) = percentage of particles finer than \( D \)
\( D \) = particle diameter

\( m \) and \( K_1 \) are constants.

**Figure 10.**—Particle-size accumulation curve of a soil sample.

Assuming the particle size accumulation curve of a soil to be as shown in Figure 10, the section of the curve representing particles with diameters in the range \( D_b-D_c \) approaches a straight line and can be represented by equation (4).

**Before sedimentation.**—Since particles of all sizes are uniformly distributed throughout the suspending medium before sedimentation, the weight of soil particles contained in the layer between level \( B-B \) and level \( C-C \) can be computed by the following equation:

\[
W = he - h_b \frac{W}{H}
\]  

where \( w \) = weight of soil contained in the layer (including particles of all sizes),
\( W \) = total weight of dispersed soil,
\( h_e - h_b \) = thickness of the layer,
\( H \) = total depth of the soil suspension in the container.

If particles with diameter smaller than \( D_b \) are considered, their weight can be determined from the equation,

\[
w_{fb} = \frac{w_{fb}}{100} \]  

where \( w_{fb} \) = weight of particles having diameter smaller than \( D_b \) contained in the layer before sedimentation,
\( P_f \) = percentage of particles having diameter smaller than \( D_b \) in the dispersed sample (Fig. 10).

If particles in the size range \( D_b-D_q \) (Fig. 10) are considered, their weight can be determined from the equation,

\[
w_{bq} = \frac{w}{100} (P_q - P_f)
\]  

where \( w_{bq} \) = weight of particles in the size range \( D_b-D_q \) contained in the layer before sedimentation,
\( P_q \) = percentage of particles finer than size \( D_q \) in the dispersed sample.

Substituting equation (4) in equation (7) and simplifying,

\[
w_{bq} = \frac{w m}{100} \log \frac{D_q}{D_b}
\]
Equations (7) and (8) are valid when $D_q$ is within the range $D_b - D_c$ because only the segment of the particle-size accumulation curve representing this size range approaches a straight line.

After a given sedimentation period. — Figure 9(b) shows that those particles with diameter smaller than $D_b$ which were at level $T-T$ before sedimentation have settled through certain distances after a given sedimentation period, but their new positions are still above level $B-B$. Particles finer than size $D_b$ are therefore still uniformly distributed throughout the layer at the instant of extraction, and equation (6) can be used to compute the weight of particles obtained from the layer.

The weight of particles in the size range $D_b - D_q$ (Fig. 9) suspended in the layer will be less than that computed by equation (7) because particles in this size range are no longer uniformly distributed throughout the layer at the end of the sedimentation period. The settling distance of particles of size $D_i$ is $h_i$ (Fig. 9). Therefore, at the end of the sedimentation period no particles of size $D_i$ remain in suspension above a level which is $h_i$ below level $T-T$. Particles of size $D_i$ in the layer between levels $B-B$ and $C-C$ remain in suspension only in the lower portion. The thickness of this lower portion is $(h_c - h_i)$, while the thickness of the layer is $(h_c - h_b)$.

The weight of soil particles of an infinitesimal size range embracing size $D_i$ which remain in suspension in the layer between levels $B-B$ and $C-C$ at the end of the given sedimentation period will be

$$\frac{h_c - h_i}{h_c - h_b} \cdot \frac{w \Delta P_i}{100}$$

where $\Delta P_i$ is the percentage by weight of the particles within the infinitesimal size range in the dispersed sample. By a process of summation, the following expression can be derived:

$$w_{bq}' = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{h_c - h_i}{h_c - h_b} \cdot \frac{w \Delta P_i}{100}$$

or

$$w_{bq}' = \frac{1}{100} \int_{0}^{h_c} \int_{h_b}^{h_c} \frac{h_c - h}{h_c - h_b} \cdot w \, dP$$

where $w_{bq}' = \text{weight of particles in the size range } D_b - D_q \text{ contained in the layer at the end of the sedimentation period. In this analysis, } w_{bq}' \text{ is also the weight of particles in the size range } D_b - D_q \text{ obtained from the layer.}$

From equation (4),

$$dP = \frac{-m \log e}{D} \, dD$$

Substituting equations (3) and (10) in equation (9),

$$w_{bq}' = \frac{u m \log e}{100 (h_c - h_b)} \int_{D_b}^{D_q} \frac{(h_c - KD^2)}{D} \, dD$$

Integrating

$$w_{bq}' = \frac{u m \log e}{100 (h_c - h_b)} \left[ h_c \ln \frac{D_q}{D_b} - \frac{K(D_q^2 - D_b^2)}{2} \right]$$

Substituting $h = K D^2 \ln D = \log e \ln D$ and simplifying

$$w_{bq}' = \frac{u m}{100 (h_c - h_b)} \left[ h_c \log \frac{D_q}{D_b} - \frac{(h_q - h_b) \log e}{2} \right]$$

(11)
From equations (8) and (11)

\[
\frac{w_{bq'}}{w_{bq}} = \frac{1}{h_c - h_b} \left[ h_c - \frac{(h_q - h_b) \log e}{2 \log \frac{D_q}{D_b}} \right]
\] 

Equation (12) gives the ratio of the weight of particles in the size range \(D_b - D_q\) obtained from the layer after a given sedimentation period to the weight of particles in the same range prior to sedimentation.

It will be noted that equations (11) and (12) are valid only when \(D_q\) is within the size range \(D_b - D_c\) and the segment of the particle-size accumulation curve representing this size range approaches a straight line. If the segment of the curve differs significantly from a straight line, a similar approach for equation derivation can be followed except that the segment of the curve should be either approximated by a number of straight line segments or represented by a nonlinear equation.

APPENDIX II

Example of Calculations

The methods of calculation for determining the size composition of clay-size material obtained by the layer and the decantation methods of extraction are practically the same. In the example which follows, only the computation relating to the layer method is presented.

The gradation of a hypothetical soil has been shown in Table 1 and Figure 3. The soil is dispersed in distilled water, and after a sedimentation period of six hours a layer positioned as in Case I (see Table 2) is removed from the soil suspension. The size composition of the material obtained from the removed layer is to be determined.

Data relating to the soil suspension have been discussed under the heading "MATHEMATICAL ANALYSIS". The pertinent values are summarized as follows:

- \(W = 150\) g
- \(H = 21.00\) cm
- \(h_b = 4.50\) cm
- \(h_c = 8.00\) cm
- \(D_b = 0.00150\) mm
- \(D_c = 0.00200\) mm
- \(P_b = 36.2\)
- \(P_c = 40.0\)

The significance of each symbol is given in Appendix I.

Before computing the weight of particles in any specific size range, the constants \(m\) and \(K_1\) must be determined. (See equation (4)). From the gradation of the hypothetical soil given in Table 1,

\[
P = 0 \quad \text{for} \quad D = 0.00010 \text{ mm}
\]

\[
P = 30.7 \quad \text{for} \quad D = 0.00100 \text{ mm}
\]

Substituting in equation (4) and solving simultaneously for the two unknowns \(m\) and \(K_1\), we obtain

\[
m = 30.7
\]

\[
K_1 = 122.8
\]

We then have for equation (4),

\[
P = 30.7 \log D + 122.8
\]

In determining the size composition of clay-size material obtained from the layer, several control sizes should be selected. By computing the weight of material finer than each control size, the particle-size accumulation curve of the extracted material
can be plotted. The control sizes used in this example are 0.00200 mm, 0.00175 mm, 0.00150 mm, 0.00100 mm, 0.00060 mm, 0.00030 mm, and 0.00010 mm.

The data in Figure 9(b) show that the material finer than 0.00150 mm is still uniformly distributed throughout the layer at the instant of extraction. The material in this size range obtained from the layer can therefore be computed by combining equations (5) and (6) to obtain

$$w_{ib} = \frac{h_c - h_b}{100} WP_b$$

Substituting $h_c = 8.00$ cm, $h_b = 4.50$ cm, $H = 21.00$ cm, $W = 150$ g, $P_b = 36.2$

we get

$$w_{ib} = 9.05 \text{ g}$$

The gradation of this material is the same as that of the fraction finer than 0.00150 mm in the hypothetical soil. The weight of material finer than 0.00100 mm obtained from the layer can then be computed as follows:

Weight of minus 0.00100 mm material = $9.05 \times \frac{30.7}{36.2} = 7.68$ g

where 30.7 and 36.2 are the percentages of material finer than 0.00100 mm and 0.00150 mm respectively in the hypothetical soil (see Table 1).

Similarly, the weights of minus 0.00060 mm and minus 0.00030 mm materials obtained from the layer can be computed and results are given in Table 4. The weight of minus 0.00010 mm material obtained from the layer will be zero because the hypothetical soil contains no particles finer than 0.00010 mm.

**Table 4. — Size Composition of Clay-Size Material Obtained from Case I Layer**

<table>
<thead>
<tr>
<th>Particle Size, mm</th>
<th>Weight of material finer than size shown, g</th>
<th>Percent of material finer than size shown</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00200</td>
<td>9.58</td>
<td>100</td>
</tr>
<tr>
<td>0.00175</td>
<td>9.45</td>
<td>98.7</td>
</tr>
<tr>
<td>0.00150</td>
<td>9.05</td>
<td>94.5</td>
</tr>
<tr>
<td>0.00100</td>
<td>7.68</td>
<td>80.2</td>
</tr>
<tr>
<td>0.00060</td>
<td>5.98</td>
<td>62.5</td>
</tr>
<tr>
<td>0.00030</td>
<td>3.68</td>
<td>38.4</td>
</tr>
<tr>
<td>0.00010</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since $D_e = 0.00200$ mm, Figure 9(b) shows that all the material obtained from the layer will be finer than 0.00200 mm. The weight of material in the size range 0.00200 - 0.00150 mm obtained from the layer can be computed by combining equations (5) and (11) to obtain

$$w_{bq} = \frac{mW}{100H} \left[ h_c \log \frac{D_q}{Db} - \frac{(h_q - h_b) \log e}{2} \right]$$

Substituting $m = 30.7$ as given in equation (13)

$W = 150$ g

$H = 21.00$ cm

$h_q = h_c = 8.00$ cm.
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\[ D_q = D_r = 0.00200 \text{ mm} \]
\[ h_s = 4.50 \text{ cm} \]
\[ D_t = 0.00150 \text{ mm} \]

We get \( w_{r_0} = 0.53 \text{ g.} \)

Hence

Weight of minus 0.00200 mm material

\[ w_{r_0} + w_{s_0} = 9.05 + 0.53 = 9.58 \text{ g.} \]

In a similar manner, the weight of material in the size range 0.00175 - 0.00150 mm obtained from the layer can be computed by using equation (14). In this example,

\[ D_q = 0.00175 \text{ mm} \]

From equation (2),

\[ h_s = 2.00 \times 10^6 D_q^2 \]
\[ = 2.00 \times 10^6 (0.00175)^2 = 6.12 \text{ cm} \]

The values of \( m, W, h, k, D, h_b, \) and \( D_b \) are the same as before. Substituting to equation (14), the weight of material in the size range 0.00175 - 0.00150 mm is found to be 0.40 g.

Weight of minus 0.00175 mm material \( = 9.05 + 0.40 = 9.45 \text{ g.} \) From Appendix I, it will be noted that the weight of the material in the size ranges 0.00200 - 0.00150 mm and 0.00175 - 0.00150 mm can also be computed by combining equations (7) and (12).

The weights of material finer than each control size are summarized in Table 4. To plot the particle-size accumulation curve for the material obtained from the layer, the size composition of the material is expressed on a percentage basis in the same table. The particle-size accumulation curve is plotted in Figure 4.