ENERGY DISSIPATION OF A KAOLINITE
AT DIFFERENT WATER CONTENTS

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Abstract — A Georgia kaolinite, at water contents from 55 to 95 per cent, was tested by means of a Weissenberg Rheogoniometer under conditions of pure shear with sinusoidally varying deformation over a frequency range of 3 decades. The results, including time-dependent effects, are expressed in terms of the magnitude of the complex modulus and the phase angle, as developed in the theory of linear viscoelasticity, and stress-strain hysteresis curves. The complex modulus is a two-component quantity, which has a real part associated with the elastic or energy storage characteristics of the material and an imaginary part associated with its viscous or energy dissipation characteristics. Although the complex modulus interpretation is very good for linearly viscoelastic materials, its applicability and usefulness diminishes as the material departs from linear viscoelastic behavior. On the other hand, the determination of energy dissipation from stress-strain hysteresis curves does not depend on any assumption concerning material behavior, because the area enclosed by the curve gives a direct measure of the energy dissipated in a single cycle of deformation. The dissipation characteristics obtained by the two methods are compared and used to illustrate the degree of validity and some limitations of linear viscoelasticity theory.

INTRODUCTION

As has been discussed by Krizek and Franklin (1967) and others, the problem of determining and specifying the rheologic response characteristics of a clay is very complicated and dependent on a multitude of parameters; in general, the mechanical behavior of a clay may exhibit both energy storage and energy dissipation response characteristics. In recent years the rapidly advancing theory of linear viscoelasticity has been often proposed to describe these rheologic behavioral characteristics. On one hand, a great deal of research effort has been given to the formulation of the mechanical behavior of clays in terms of viscoelastic parameters; on the other hand, the applicability of linear viscoelastic theory to a material such as clay has been questioned by many researchers.

The work reported herein is directed toward studying the energy dissipation characteristics of a particular kaolin clay over a range of water contents. Energy losses are measured directly, without the assumption of a constitutive relation for the clay, and compared with values calculated by using the theory of linear viscoelasticity in conjunction with measured phase angles. The results obtained, together with their interpretation, are discussed and compared with related work by other investigators.

THEORETICAL CONSIDERATIONS

Energy dissipation

When materials are set in vibration, some of the input energy is dissipated by the various mechanisms which may collectively be termed internal friction. For liquids and gases these dissipative mechanisms are generally attributed to viscosity and thermal conduction, and they may be treated analytically with reasonable degrees of accuracy. However, the energy dissipation characteristics of solids are found to be much more complex and to vary considerably with the nature of the solid; there is at present (1968) no generally accepted theory of energy dissipation in solids, and more experimental data are required.

There are several indirect methods of defining the energy dissipation of a material, and most of these depend on the assumption that the restoring forces are proportional to the amplitudes of vibration while the dissipative forces are proportional to the velocities. For example, when these conditions apply, the logarithmic decrement, which is defined as the natural logarithm of the ratio between amplitudes of successive free oscillations, may be taken as a measure of the energy dissipation. Alternatively, another indirect measure of energy dissipation is given by its inverse relation to the sharpness of the resonance curve under
forced vibration. Still another measure of dissipated energy may be obtained from the phase difference between sinusoidally applied stresses or strains and measured response. However, each of these indirect measures is based on the assumption stated, namely, that the material obeys a linear viscoelastic constitutive relation. Under such conditions, these measures of energy dissipation are not unrestricted, and this fact must be remembered in their application. Nevertheless, linear viscoelastic theory can provide insight into the analysis of experimental data on clays, even though, in general, they do not truly exhibit linear viscoelastic stress-strain-time behavior, and these techniques are being pursued with some degree of success.

The most direct method of defining energy dissipation is by measuring the actual energy dissipated in taking a specimen through a deformation cycle. The ratio of the energy dissipated for a single cycle of deformation to the total elastic energy stored in a perfectly elastic specimen when the strain is a maximum is called the specific damping capacity or the specific loss of the material. This parameter can be measured directly without the use of any assumptions regarding the constitutive response of the material or the nature of the internal friction. However, the quantitative value will generally depend on the stress and strain amplitudes, frequency, past history, etc. More detailed discussions of the indirect methods for determining energy dissipation are given by Ferry (1961), Kolsky (1963), and others.

When a material is subjected to an oscillatory applied stress or strain, a plot of stress-strain values for corresponding times throughout a given cycle will yield a hysteresis loop. Typical hysteresis loops produced by various ideal materials subjected to a harmonically applied stress or strain are shown in Fig. 1. Although hysteresis loops may be produced for a variety of loading paths, only those produced by harmonically applied strains will be considered herein. The use of harmonically applied stresses or strains draws its advantage from the greatly simplified theoretical analysis which results in the case of linear viscoelastic materials; however, researchers in the field of clay rheology have, in general, not yet exploited the potential of this means of determining dynamic clay properties. The energy dissipated for a single cycle of deformation is given by the area enclosed by the loop. As can be seen from Fig. 1a, no energy is lost in a stress-strain cycle of a perfectly elastic material; however, Fig. 1e shows that the energy loss in a perfectly viscous material, for which stress and strain are in quadrature, is \(\pi r_0 y_0\). The characteristic loops given in Figs. 1b, 1c, and 1d for various ideal elastic-plastic materials are parallelograms, while the loop for a linear viscoelastic material, as shown

![Hysteresis loops for ideal materials subjected to harmonic deformations.](image-url)
in Fig. 1f, is an ellipse. In general, real materials (especially clays) do not manifest exactly any of these characteristic patterns, but, for the sake of analysis, their response is often assumed to be described by one of the idealizations shown.

Viscoelastic relations

Detailed developments of linear viscoelastic theory have been presented by Gross (1953), Bland (1960), Ferry (1961), and others, and an interpretation in light of this theory of the mechanical response of a soft clay was presented by Krizek and Franklin (1966). Although these details will not be repeated herein, the appropriate relations utilized in the further development of this work will be summarized in Eqs. 1–5, which define the complex viscoelastic shear parameters (storage modulus \( G' \), loss modulus \( G'' \), complex modulus \( G^* \), magnitude of complex modulus \( |G^*| \), and phase angle \( \delta \)) for a linear material which is subjected to a sinusoidally varying shear strain of amplitude \( \gamma_0 \) at a given frequency of oscillation \( \omega \) and whose response is a sinusoidally varying shear stress of amplitude \( \tau_0 \) at the same frequency but leading the strain by a phase angle \( \delta \).

\[
G' = \frac{\tau_0 \cos \delta}{\gamma_0} \quad (1)
\]

\[
G'' = \frac{\tau_0 \sin \delta}{\gamma_0} \quad (2)
\]

\[
G^* = G' + iG'' \quad (3)
\]

\[
|G^*| = \frac{\tau_0}{\gamma_0} \quad (4)
\]

\[
\delta = \tan^{-1} \frac{G''}{G'} \quad (5)
\]

For the conditions just described, the imposed strain and resulting stress can be expressed as

\[
\gamma(t) = \gamma_0 \cos \omega t \quad (6)
\]

and

\[
\tau(t) = \tau_0 \cos (\omega t - \delta) \quad (7)
\]

The energy dissipated in a single cycle of deformation is given by

\[
\Delta W = \int_0^{2\pi/\omega} \tau \frac{dy}{dt} \, dt \quad (8)
\]

or

\[
\Delta W = \int_0^{2\pi/\omega} \omega \gamma_0 \tau \sin \omega t \cos (\omega t - \delta) \, dt \quad (9)
\]

which upon integration and evaluation yields

\[
\Delta W = \frac{\pi \gamma_0 \tau_0 \sin \delta}{2} \quad (10)
\]

An equivalent expression, using the magnitude of the complex modulus given by Eq. 4, is

\[
\Delta W = \pi \gamma_0 |G^*| \sin \delta \quad (11)
\]

The maximum strain energy stored in a perfectly elastic material at the same amplitudes is

\[
W = \frac{1}{2} \gamma_0 \tau_0 \quad (12)
\]

The specific loss is defined as \( \Delta W/W \), and it is equal to \( 2\pi \sin \delta \) for a linearly viscoelastic material.

If stress is plotted against strain over a single cycle of deformation for a linear viscoelastic material, the hysteresis loop will be an ellipse, as shown in Fig. 1f. The axes of the ellipse are, in general, at some angle to the coordinate axes. The relation between the phase angle and the dimensions of the ellipse is given by

\[
\sin \delta = \frac{ab}{XY} \quad (13)
\]

where \( a \) and \( b \) are the semiminor and semimajor axes of the ellipse, and \( X \) and \( Y \) are the horizontal and vertical dimensions corresponding to \( \gamma_0 \) and \( \tau_0 \), respectively.

MATERIAL INVESTIGATED

The clay used in this investigation was a water-washed Georgia kaolinite with a grain size distribution wherein approx. 97 per cent of the particles are smaller than \( 5 \mu \) and 60 per cent are smaller than \( 1 \mu \). Some of the physical characteristics of this clay are: liquid limit 53 per cent, plastic limit 35 per cent, shrinkage limit 32 per cent, and specific gravity 2.61. The X-Ray diffraction and differential temperature graphs indicate that kaolinite is the only mineral present in significant quantity.

The preparation of test specimens was accomplished by manually mixing powdered clay with distilled water to obtain an aqueous clay mixture with a predetermined nominal water content; after being mixed as thoroughly as possible by hand, the mixtures were sealed in air-tight containers and stored in a high humidity room for a period of approx. 4 months before they were tested.

TEST APPARATUS AND INSTRUMENTATION

Test apparatus

The tests described herein were performed on a Weissenberg Rheogoniometer with a cone-plate sample holder. A description of this device, together with the advantages it offers by producing a
relatively homogeneous state of shear strain on the specimen, were discussed by Krizek and Franklin (1967). The cone and plate which were used for these tests had a diameter of 5 cms and a cone angle of 6°. The torsion head assembly with this cone, the transducer mount, and the particular torsion bar used had a natural frequency of about 158 c/s. Since the frequency range of this experimental program was from about 0.01 c/s to 10 c/s, measured response should not be affected noticeably by the natural frequency of the system.

**Instrumentation**

Various aspects of the response characteristics of the clay sample and test apparatus, including determination of the correct gap setting for the truncated cone-plate assembly, strain amplitude control, resulting load and deformation of the specimen, etc., were obtained by means of a standard set of auxiliary equipment, including transducers, meters, and amplifiers, manufactured by the Boulton Paul Aircraft Company of England (the Rheogoniometer is an English-made device). The load-deformation (ultimately converted to stress-strain) response was recorded by means of an Offner Dynograph (Model RS, heat sensitive, rectilinear recording); this provided satisfactory response, including phase angle measurement, for frequencies up to 10 c/s.

**Calibration**

The Rheogoniometer and its recording system were checked for accuracy and calibration by measuring the stress-strain response of a National Bureau of Standards standard viscosity oil OB-36. For a Newtonian oil over the frequency range under consideration, harmonically applied stresses or strains should result in harmonically measured strains or stresses in quadrature; hence, the resulting hysteresis loop, when plotted to appropriate horizontal and vertical scales, should be a circle as shown by the dashed curve in Fig. 2. The experimentally measured response of the oil at the ambient environmental conditions is indicated by the data points and solid curve in the figure. These curves provide a measure of the distortion introduced by the instrumentation into the measured response. In particular, they indicate that a small amount of distortion in the shape of the loop is produced by nonlinearity in the electronic components of the instrumentation; however, the error introduced into the measurement of the total area of the loop is small. The distortion introduced by the instrumentation nonlinearity also manifests itself in a slight imperfection of symmetry in the hysteresis loops for the clay samples.

**EXPERIMENTAL PROCEDURE**

**Conduct of test**

Rao and Nagaraj (1967) discuss the influence of initial soil structure on shear strength characteristics of a saturated kaolinite clay subjected to vibratory loading of a type different from that used herein, and they conclude that initial soil structure may play an important role. The nature of the clay fabric and residual stresses which may exist in the sample as a result of the placement technique is unknown; however, the same procedure was followed consistently. After an excess of the clay-water mixture was placed on the plate, the cone was immediately lowered into the operating position and the excess clay was trimmed. All tests were conducted in a temperature-controlled room at a temperature of 23 ± 1°C, and the sample containers were placed in the room several days before testing to insure that they attained room temperature.

All specimens were subjected to a periodic torsional shear deformation by oscillating the cone platen at a chosen frequency. The upper platen was fixed to a calibrated torsion bar, and the angle of twist of the torsion bar was measured by a transducer located at the tip of a radial arm fastened to the torsion bar. The relative displacements of the two platens were measured by a transducer with its core and field casing fixed in custom-made clamps connected directly to the platens. In this way the relative displacement between the two platens, and hence the strain, could be measured directly in true phase, and corrections to account for the twist of the torsion bar did not have to be applied. This is an improvement over the technique previously reported by Krizek and Franklin (1967).

For the tests reported herein, the amplitude of the input oscillation (angle of twist of the lower platen) was held constant while the frequency of oscillation was varied during the test. With one exception, all tests reported herein were conducted at a strain amplitude of 0.00384 ± 7 per cent. The test frequencies were obtained by varying the gear settings on the oscillatory drive, and results were recorded at discrete values of frequency between 0.0095 c/s and 9.5 c/s. Each test condition was maintained until transient effects had virtually disappeared and a reasonably steady-state condition was obtained. The imposed deformation and stress response were recorded on a two channel oscillograph, and magnitudes for each of the respective parameters were recorded from these records. At the conclusion of the test, the sample was removed from the platen, weighed, and dried for a water content determination.
Values of the phase difference $\Delta t$ were measured from the oscillograph records as the lag between zero axis crossings of the stress and strain curves; with the frequency of oscillation known, the phase angle $\delta$ can be calculated from the relation $\delta = \omega \Delta t$. If the time differences between peaks were used, measured phase differences would be much smaller than those measured between zero axis crossings, because the stress curves in the non-linear response region of the clay are not symmetrical. It was felt that the relations between zero axis crossings would yield more meaningful expressions of the energy dissipation characteristics of the material than relations between stress and strain peaks. Values of energy dissipation and specific loss were calculated from these phase angles by using the viscoelastic relations given previously.

In order to plot a hysteresis loop, as shown in Fig. 3, values were picked from the stress and strain curves at an average of approximately 20 points over a single cycle of deformation. It was found that this procedure was practical for frequencies up to 9-5 c/s. The areas enclosed by these hysteresis loops (or stress versus strain curves) were measured with a planimeter, and these areas represent the energy dissipated during a single cycle of deformation. A second value for the specific loss was obtained from the ratio of the energy dissipated to the quantity $\gamma_0^2$, which represents the maximum stored energy in a perfectly elastic material at the same stress and strain amplitudes.

**EXPERIMENTAL RESULTS**

**Interpretation**

Because the experimental response shown in Fig. 3 does not match precisely that of any of the ideal materials whose hysteresis loops are shown in Fig. 1, caution is recommended regarding the unsubstantiated assumption that clay response may be described by one of these idealizations. In order to obtain an appreciation for the degree to which the measured hysteresis response may be approximated by the response of a linear viscoelastic material with the same stress and strain amplitudes, the elliptical hysteresis loop associated with the
latter material and calculated from the measured phase angle by use of Eq. 13 is superimposed on the experimental loop in Fig. 3.

Figure 4 shows a typical series of nine normalized hysteresis loops which are associated with nine discrete moisture contents and a frequency of oscillation of 0.95 c/s. The similarity in the shape of these loops suggests weak dependence on water content over the range investigated. The slight variations in the shapes of the loops may be a manifestation of the nature of the energy dissipation in the clay, as well as possible experimental error.

Typical response data for the magnitude of the complex modulus and the phase angle as functions of water content for a constant frequency of oscillation are shown in Figs. 5 and 6, respectively. As indicated by Eq. 11, these parameters may be used to calculate the energy loss in the material. While the magnitude of the complex modulus exhibited a strong exponential dependence on water content, the phase angle was found to be essentially independent of water content over the range tested; similar behavior was observed at other frequencies in the test program.

The energy loss per cycle, $\Delta W$, for a series of ten constant-frequency hysteresis loops, which are shown in Figs 3 and 4, is determined by measuring the area of the loop, and these values are plotted against water content in Fig. 7. Included in this figure are the energy losses determined for each of the cases independently of the hysteresis loops by use of Eq. 11 and the measured parameters given in Figs 5 and 6. Although these values differ slightly from those obtained from the hysteresis loops, they are nevertheless in reasonably close agreement and suggest that, despite the discrepancies already noted, linear viscoelastic theory can provide an insight into more complicated material behavior, and energy losses for the test conditions described can be approximated to a reasonable degree of accuracy by assuming the clay to be a linear viscoelastic material and utilizing the extensive theory already available in other fields. However, caution must be exercised against over-generalizing, because material characteristics other than energy dissipation may not lend such close agreement. Also, the observed response may be peculiar to the particular clay tested.

Figure 8 shows a plot of specific loss, $\Delta W/W$, versus water content. It is seen that normalization of the energy loss $\Delta W$ by the parameter $W$ tends to
minimize or eliminate the water content dependence manifested in Fig. 7. This water content dependence is, of course, implicitly contained in the parameter $W$, which is in turn a function of the stress and strain amplitudes, as given by Eq. 12.

The test program included ten series of tests, as just described, four of which were at seven discrete values of frequency over a range of three decades from 0.0095 to 9.5 c/s. The remainder were at five values of frequency from 0.095 to 9.5 c/s. Average values of the specific loss for the range of water contents tested, such as determined from Fig. 8, are plotted in Fig. 9 for both the hysteresis loop determination and the phase angle determination procedures. Although the degree of frequency dependence was found to be a function of the method used to determine the energy loss, the maximum variation between values obtained by the two techniques was approximately 10 per cent.

**Discussion**

Several investigators have used testing procedures, methods of analysis, or materials related to those employed herein to investigate the energy dissipation characteristics of soils, and their work will be discussed briefly in light of the results of this study. However, it should be emphasized that none of their studies are precisely parallel to the one reported in this paper, and the relative advantages and disadvantages of each investigation, as well as the assumptions required and the validity of the results, should be thoroughly considered.

As one approach to predict the peak stress attenuation in cohesive soils, Seaman and Whitman...
(1964) and Seaman (1966) postulated a linear viscoelastic constant \(\tan \delta\) model. Such a model generates a constant hysteresis loop for all frequencies; however, this loop may vary with water content, or consistency, changes. Hampton and Wetzel (1966) used this model with selected values of 0.2 and 0.4 for \(\tan \delta\) to compare theoretical and experimental results for peak stress attenuation in an Edgar Plastic Kaolin (EPK) clay. Water contents for this work were in the neighborhood of 30 per cent. For the experimental results in their study, they concluded that the constant \(\tan \delta\) model appeared to be reasonably representative if \(\delta\) were properly evaluated, and the major problem lies in arriving at the appropriate value for \(\delta\).

Parmelee et al (1964) conducted a series of cyclic load tests on a soft clay for the purpose of determining the variation of its deformation and
damping characteristics with depth. The clay deposit exhibited average water contents between 50 and 80 per cent, although strengths at any given elevation were found to vary by ±50 per cent from the average value. Cyclic load tests consisting of 10 cycles and using a constant stress rate of 0.2 kg/
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Average Values for All Water Contents at a Given Frequency

\[ A' = \frac{\Delta w}{\Delta W} \]

Measured by Hysteresis Loop

Measured by Phase Angle

Fig. 9. Specific loss versus frequency.

Energy dissipation measured from hysteresis loops and formulated in terms of specific loss was generally slightly different than that calculated from the phase angle approach; however, despite the presence of plastic components in the response characteristics, the magnitude of the difference indicates that the assumption of linear viscoelasticity (which is inherent in the phase angle approach) may be a reasonably good first-order approximation for the constitutive behavior of a soft clay.

CONCLUSIONS

Based on the results of this study on a particular soft kaolin clay, the following conclusions may be drawn:

1. At a given frequency of oscillation, the specific loss is relatively independent of water content over the range tested, which was entirely above the liquid limit of the clay.

2. Energy dissipation measured from hysteresis loops and formulated in terms of specific loss was generally slightly different than that calculated from the phase angle approach; however, despite the presence of plastic components in the response characteristics, the magnitude of the difference indicates that the assumption of linear viscoelasticity (which is inherent in the phase angle approach) may be a reasonably good first-order approximation for the constitutive behavior of a soft clay.

3. Although these results indicate that the phase angle and specific loss do vary with frequency in a systematic manner, the magnitude of the variation is small over the frequency range tested, and the constant \( \tan \delta \) model used by some investigators seems reasonable.

4. The technique of applying harmonic stresses or strains to a clay specimen and measuring its response seems to provide a potentially fruitful, but virtually unexploited, approach to determining certain rheologic clay parameters.

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Résumé—On a fait des expériences sur une kaolinite géorgienne, à teneur d'eau de 55 à 95% à l'aide d'un Rhéogoniomètre Weissenberg sous des conditions de cisaille pure avec déformation de variation sinusoidale dur une gamme de fréquence de trois décennies. Les résultats, y-compris les effets tributaires du temps, sont exprimés par rapport à la taille du module complexe et de l'angle de phase, tels qu'ils sont développés dans la théorie de la viscoélasticité linéaire, et les courbes de hystérésis par suite des diverses tensions. Le module complexe est une quantité à deux composantes, dont la partie réelle s'associe aux caractéristiques d'elasticité ou de conservation d'énergie de la matière, alors que la partie imaginaire s'associe aux caractéristiques de viscosité ou de dissipation d'énergie. Bien que l'interprétation du module complexe soit excellente pour les matières à viscoélasticité linéaire, elle devient de moins en moins applicable et utile à mesure que la matière s'éloigne d'un comportement viscoélastique linéaire. Cependant, la détermination de la dissipation d'énergie par des courbes d'hystérésis par suite des diverses tensions ne dépend pas, elle, d'hypothèses quant au comportement de la matière, puisque la surface englobée par la courbe donne une mesure directe de l'énergie dissipée au cours d'un cycle unique de déformation. Les caractéristiques de dissipation obtenues grâce aux deux méthodes sont comparées et servent à illustrer le degré de validité et certaines des limitations de la théorie de la viscoélasticité linéaire.


Резюме—Джорджия-каолинит, с водосодержанием от 55 до 95%, был исследован пользуясь реогониометром Вейсенберга в условиях чистого среза с синусоидально меняющейся деформацией в частотном диапазоне 3 декад. Результаты, включая зависящие от времени эффекты, выражаются как величина комплексного модуля и фазового угла по теории линейной вязкоупругости и как кривые гистерезиса зависимости деформации от напряжения. Комплексный модуль это двухкомпонентная величина, которая имеет настоящую часть, связанную с
характеристиками упругости или хранения энергии материала, и мнимальную часть, связанную с характеристиками вязкости и рассеяния энергии. Хотя толкование комплексного модуля и есть очень хорошим для линейно вязкоупругих материалов, применимость и пригодность его понижается по мере того, как материал отходит от своего линейного вязкоупругого поведения. С другой стороны, определение рассеяния энергии на основании гистерезисных петель зависимости деформаций от напряжения не является результатом какого-либо предположения о поведении материала, так как площадь ограниченная кривой дает непосредственный отсчет энергии, рассеянной в одном цикле деформации. Характеристики рассеяния, установленные этими методами, сравниваются и применяются для того, чтобы иллюстрировать степень пригодности и некоторые ограничения теории линейной вязкоупругости.